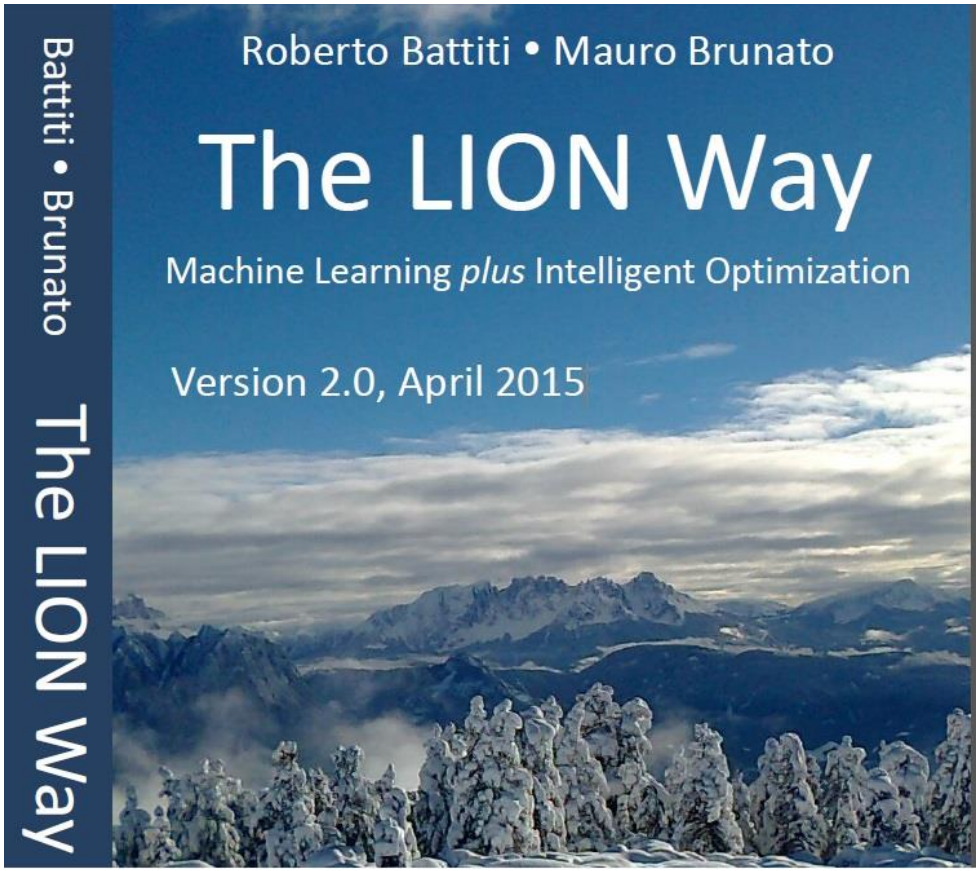


ROBERTO BATTITI, MAURO BRUNATO.
The LION Way: Machine Learning plus *Intelligent Optimization*.
LIONlab, University of Trento, Italy,
Apr 2015

http://intelligent-optimization.org/LIONbook

Text and web mining – part II

Wholly new forms of encyclopedias will appear, ready made with a mesh of associative trails running through them, ready to be dropped into the memex and there amplified.
(Vannevar Bush, 1945)

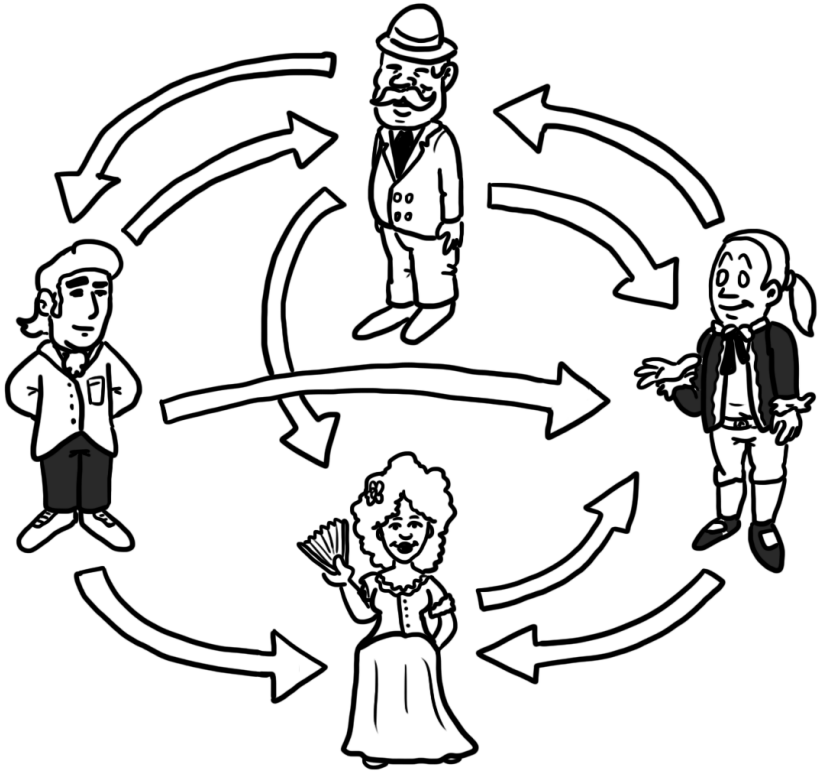
Using hyperlinks to **rank** web pages

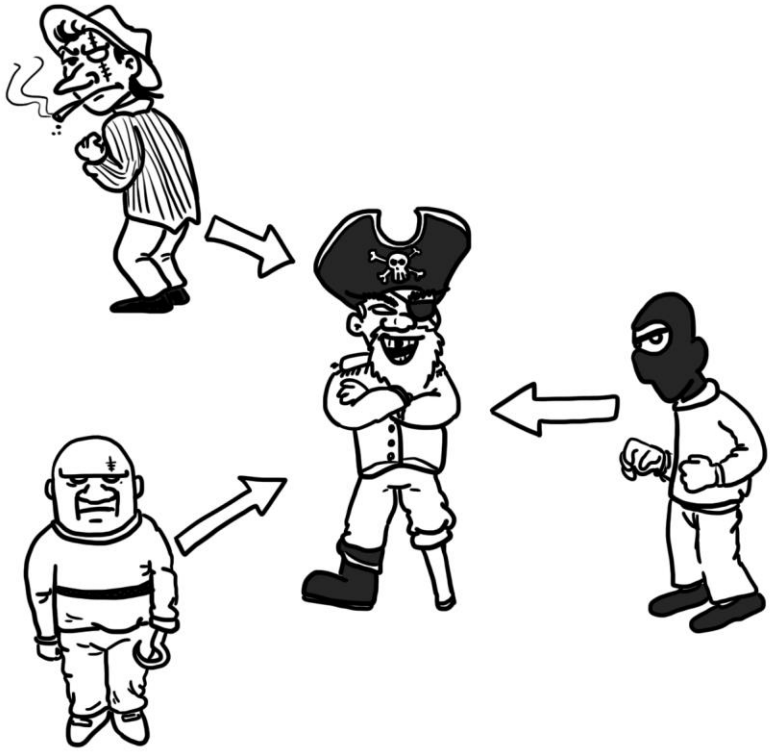
- Problem: given a query, how to retrieve a set of **high quality** and **relevant** pages from the Web.

- In scientific communities, a paper is considered of good quality if it is **cited** by other good quality papers (citation analysis in **Bibliometrics**)

- Analogy: a candidate for employment is valued if many other valued people are prepared to recommend him

Using hyperlinks to rank web pages (2)





Prestige in social networks: recommendations from (or relationship with) high-rank individuals (above) are more effective to reach a high rank than recommendations by low-rank ones (below).

Using hyperlinks to rank web pages (3)

- After a seminal paper by Marchiori about the importance of *hyper-information* (information in the hyperlinks), Larry Page and Sergey Brin developed the **PageRank** algorithm
- Same social networks principles, by substituting “recommendations” and “citations” with hyperlinks

Using hyperlinks to rank web pages (4)

- The **prestige** of a page is related to how many pages of **prestige** link to it.
- Note the **recursive definition**: to calculate the prestige, one needs to start from prestige values of other pages, and so on…
- Start with initial prestige values (random?) and **iterate**, if prestige values **converge** the problem is solved.

Using hyperlinks to rank web pages (5)

- What guarantee that the process **converges**, hopefully to the same limiting distribution, **not depending on the initial distribution** of values?

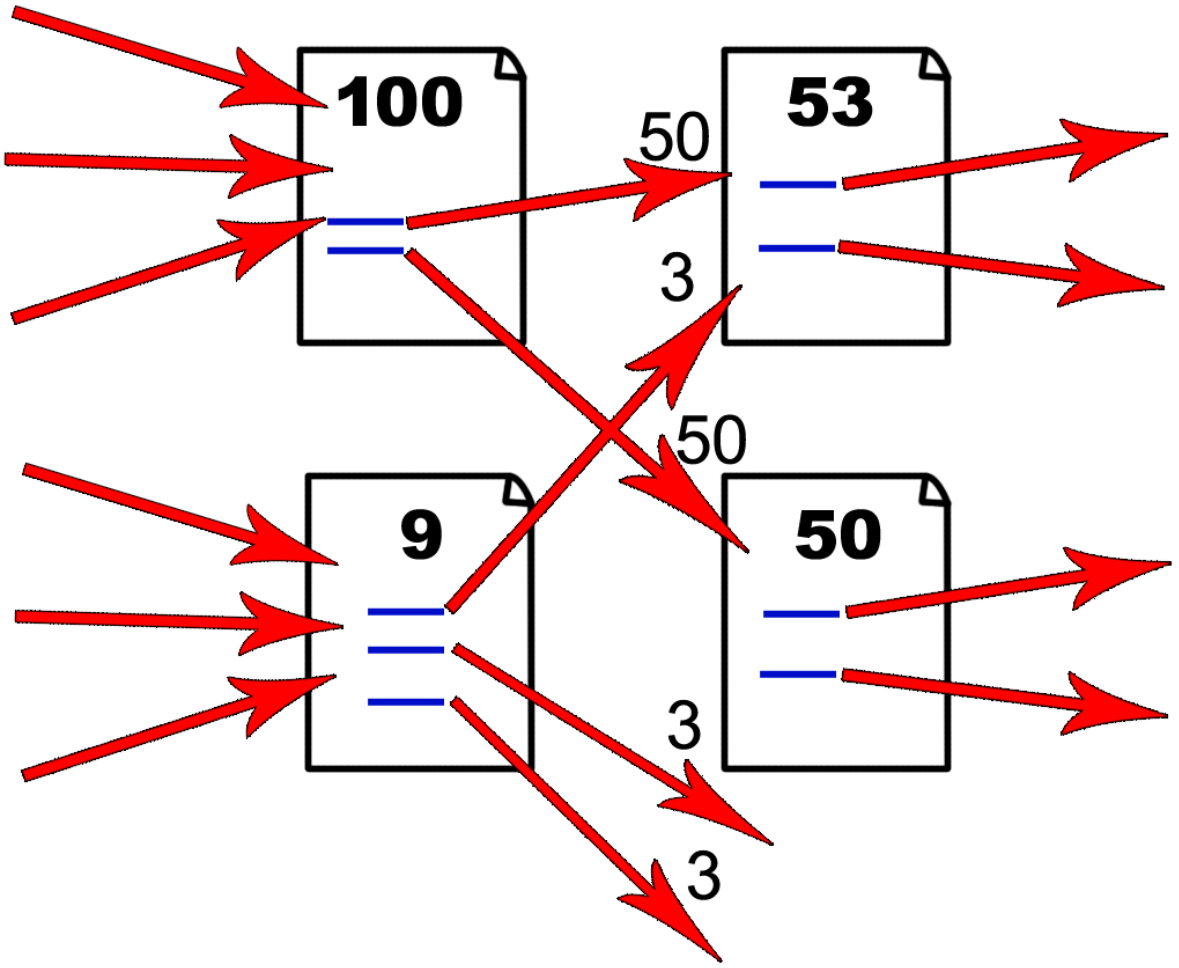
- The solution to this problem is related to basic **linear algebra** concepts of **eigenvalues** and **eigenvectors**, as well as Markov chains.

Using hyperlinks to rank web pages (6)

- Let's use **linear** definitions

- To calculate rank of a page:
  - Examine **incoming links** (the hyperlinks of other pages pointing to the given page).
  - Each incoming link from page i contributes an **addendum** equal to the rank of i divided by the number of i's outgoing links

Using hyperlinks to rank web pages (7)



Recalculating the rank of a page in PageRank**. The initial rank is distributed along the outgoing links** (adapted from the original paper).

Using hyperlinks to rank web pages (8)

- New rank values $\mathbf{p}_k$ at iteration k are obtained by a **linear transformation** of the previous values through a matrix M (derived by analyzing hyperlinks)

$$\mathbf{p}^k = M\mathbf{p}^{k-1}$$

- After k recalculations:

$$\mathbf{p}^k = M^k\mathbf{p}^0$$

Using hyperlinks to rank web pages (9)

- Assume a **basis of eigenvectors** of M exists
- let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the n **eigenvalues**, and **v**1; **v**2; …; **v**n the corresponding **eigenvectors**

  M **v**i = λi **v**i
- λ1 is the dominant eigenvalue, so that |λ1| > |λ j| for j > 1.
- The initial vector **p**0 can be written as a linear combination of the basis vectors:

$$\mathbf{p}^0 = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n.$$

Using hyperlinks to rank web pages (10)

- By linearity and definition of eigenvectors:

$$
\begin{aligned}
M^k \mathbf{p}^0 &= c_1 M^k \mathbf{v}_1 + c_2 M^k \mathbf{v}_2 + \cdots + c_n M^k \mathbf{v}_n \\
&= c_1 \lambda_1^k \mathbf{v}_1 + c_2 \lambda_2^k \mathbf{v}_2 + \cdots + c_n \lambda_n^k \mathbf{v}_n \\
&= c_1 \lambda_1^k \left(\mathbf{v}_1 + \frac{c_2}{c_1} \left(\frac{\lambda_2}{\lambda_1} \right)^k \mathbf{v}_2 + \cdots + \frac{c_n}{c_1} \left(\frac{\lambda_n}{\lambda_1} \right)^k \mathbf{v}_n \right)
\end{aligned}
$$

$\left(\frac{\lambda_2}{\lambda_1}\right)^k \to 0$ and $\left(\frac{\lambda_n}{\lambda_1}\right)^k \to 0$

- All terms tends to zero apart from the one proportional to the **dominant eigenvector**.
- A simple **iteration of matrix multiplication,** after starting from almost arbitrary initial conditions extracts the dominant eigenvector!
- **Power method** for obtaining the dominant eigenvector by the *power* of a matrix: M^k)

A surprising connection with web surfing

- Modeling the movement of a **web surfer** on the various web pages
- Let E be the adjacency matrix of the web:

  $(u, v) \in E$ (or $E_{uv} = 1$) if and only if there is a link from page u to page v.
- What is the probability p_v^1 of the surfer being at page v after one step?
- Let $N_u = \sum_v E_{uv}$ be the out-degree of page u
- After defining $L_{uv} = \frac{E_{uv}}{N_u}$

A surprising connection with web surfing (2)

- One obtains:

$$p_v^1 = \sum_u L_{uv} p_u^0 \qquad \text{or} \qquad \mathbf{p}^1 = L^T \mathbf{p}^0$$

- After i steps: $\mathbf{p}^i = L^T \mathbf{p}^{i-1}$ (again, iteration of matrix multiplication)
- If E is irreducible and aperiodic, **p** converges to the largest eigenvector

$$\lim_{i \to \infty} \mathbf{p}^i = \mathbf{p}$$

- the prestige (rank) of a page can be interpreted also as **probability that a random surfer** following links **will be found at a given page**

A surprising connection with web surfing (3)

- Real-world transition matrices: Web is not strongly connected, and that **random walks can be trapped** into cycles.
- "damping factor" corresponding to a user with an **arbitrary probability d of going to a random page** (even unconnected) at every step
- The transition becomes (**1** is the identity matrix)

$$\mathbf{p}^i = \left((1-d)L^T + \frac{d}{N}\mathbf{1}_N \right) \mathbf{p}^{i-1}.$$

A surprising connection with web surfing (4)

- The eigenvector corresponding to the largest eigenvalue can be obtained:
  - Start with random vector $\mathbf{p} \leftarrow \mathbf{p}^0$;
  - repeat:
    - update vector:

$$\mathbf{p} \leftarrow \left((1-d)L^T + \frac{d}{N}\mathbf{1}_N \right) \mathbf{p};$$

    - from time to time, normalize it:

$$\mathbf{p} \leftarrow \frac{\mathbf{p}}{\|\mathbf{p}\|_1}.$$

Normalization to avoid very large components and numerical problems. One is not interested in absolute prestige values but in relative ones

Identifying hubs and authorities: HITS

- A different analysis of the web. In a scientific community good articles are either **seminal** (i.e., many others reference to them) or **surveys** (i.e., they reference to many others).
- In the web. pages may be **authorities** or **hubs**. For example portals are very good hubs.
- Two score measures, called hubness and authority:

$$\boldsymbol{h} = (h_u), \qquad \boldsymbol{a} = (a_u)$$

Identifying hubs and authorities: HITS (2)

- HITS algorithm (Hyperlink-Induced Topic Search)
- Given query q, let Rq be the **root set** returned by an IR system. The computation is performed only on this result set.
- The **expanded set** is formed by adding all nodes linked to the root set:

$$V_q = R_q \cup \{u : ((u \to v) \vee (v \to u)) \wedge v \in R_q\}.$$

- Let Eq be the induced link subset, Gq = (Vq;Eq).

Identifying hubs and authorities: HITS (3)

- Authority and hub values are defined in terms of one another
- **hub score** h_u proportional to sum of referred authorities, **authority score** a_u proportional to the sum of referring hubs

$$\begin{aligned} \boldsymbol{a} &= E^T \boldsymbol{h} \\ \boldsymbol{h} &= E\boldsymbol{a}. \end{aligned}$$

Identifying hubs and authorities: HITS (4)

- The iterated method in HITS:
  - initialize $\boldsymbol{a}$ and $\boldsymbol{h}$ (e.g., uniformly) ;
  - repeat:
    - $\boldsymbol{h} \leftarrow E\boldsymbol{a}$;
    - $\boldsymbol{a} \leftarrow E^T\boldsymbol{h}$;
    - normalize $\boldsymbol{h}$ and $\boldsymbol{a}$.
- The top-ranking authorities and hubs are reported to the user.
- The principal eigenvector identifies the largest dense **bipartite subgraph**.

Clustering in web mining

- During web search, to avoid overloading the user, **identify groups** of closely related documents, show only a small number of representative **prototypes**.
- Queries can be ambiguous. For example, *star*: movie stars, or celestial objects?
- Mutual **similarities** in term vector space can help clustering.

Clustering in web mining (2)

- Retrieved pages can be characterized either **internally** by some intrinsic property (e.g., terms contained, coordinates in TF-IDF space) or **externally** by a measure of distance between pairs. Examples are: Euclidean distance, dot product, Jaccard coefficient.

- After defining the metric, the usual **bottom-up or top-down clustering** techniques can be used (see chapter on clustering).

Gist

- The Web is a vast expanse of data, some of it structured, some partially structured or not at all.

- **Crawling and indexing** are systematic methods to visit web pages, harvest the information contained therein and **prepare data structures for searching**, information retrieval and **ranking**.

- By **transforming text into vectors of data** (e.g., frequencies of selected words as in the vector-space model) some traditional ML techniques can be reused, but the **richer amount of structure in web documents** permits amore focused analysis.

Gist (2)

- Web-mining find explicit relationships between documents (web **links**), infer implicit ones (by **clustering**), **rank** the most relevant pages or identify the most relevant and well-connected persons in a network of people.

- **Abstraction** helps to use similar tools for networks of pages and networks of people. As a notable example, the use of **hyperlinks and linear algebra tools (eigenvectors and eigenvalues)**, previously used to rank researchers in bibliometrics, leads to a very powerful technique to rank web pages, now at the basis of Google search-engine technology.